The backstepping control of a class of random uncertain linear second order system with single known control direction

Zhaorui Ma[1], Cheng Gan[1], Na Li[2], Xia Li[1], Ruiqi Wang[3], Junwei Lei[4*]

[1] *College of Computer and Communication Engineering, Zhengzhou University of Light Industry, Zhengzhou 450002, China*

[2] *College of Electronic Information Engineering, SIAS International University of ZZU, Zhengzhou 451100, China*

[3]*Department of Armament Engineering, Naval Aeronautical and Astronautical University, Yantai Shandong 264001, China*

[4]*Department of Control Engineering, Naval Aeronautical and Astronautical University, Yantai Shandong 264001, China*

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Abstract

A kind of backsteeping control method is researched for a class of random uncertain linear two order system with single known control direction. The rightness of backstepping control is proved by constructing a Lyapunov function and it is compared with PID method. Simulation result shows that the backstepping method has a strong robustness than PID control for second order system with whole random model parameters.

Keywords: *Backstepping control, PID, Uncertainty, Second order system*

1 Introduction

Second order system has very complex dynamics and many complex engineering objects such as missile, airplane and rocket, can be viewed as a second order system for designers. So research on second order system is meaningful and enough [1-5]. Model uncertainties always exist and they are caused by environment changes or random reasons [6-8]. Backstepping method is a kind of feedback control method and it is based on Lyapunov stability theory. It is an effective and robust method for coping with system uncertainty by constructing control law step by step cleverly [9-11].

There are also some disadvantages for backstepping method. First one is that the derivative of virtual control law will be more and more complex as the order of the system increase [12-17]. It is also called differential bomb. A kind a dynamic surface method was used in some papers to solve this problem. But it is not a kind accurate method to solve the derivative of virtual signal. The main principle is to use filter to get approximate derivative. The second problem is that it is difficult for the backstepping design method to solve uncertain systems. So many papers used adaptive method with backtepping method together to solve uncertainties of system [18-25].

Adaptive backstepping controllers were designed for nonlinear models in many papers. The uncertainties in missile pitch plane's model considered in [20] are consisted of uncertain parameters and unknown nonlinear functions, where the unknown functions represents the model error or the time varying of the system. The main assumption is that these unknown functions satisfy so-called triangular bounds conditions. In particular, the unknown nonlinearities satisfy some growth conditions characterized by bounding functions composed of known functions multiplied by unknown parameters. Polycarpou & Ioannou [21] designed an adaptive backstepping controller which guaranteed the uncertain system's uniform ultimate boundedness. Based on that, Seung-Hwan Kim & Yon-Sik Kim [22]further estimated a virtual control coefficient.

In this paper, a kind of second order random system is researched and a new kind of backstepping control law is designed. To avoid the differential bomb problem, the virtual control of the first order system was designed to be very simple so it is easy to solve its derivative. And a kind of adaptive method is adopted to solve the unknown parameter problem of uncertain random systems.

Also the detailed numerical simulations were done by using both PID control method and backstepping method. And comparison between the above two methods are done. Analysis result shows the rightness and good performance of the backstepping control law.

2 Problem Description

Second order system with single control direction is a special situation. The control direction is defined as the coefficient of control u . The system model can be described as

* Corresponding author's E-mail: leijunwei@126.com

$$\dot{x} = Ax + bu\,, \tag{1}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},\ b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Parameter of model is totally unknown and A is a rand matrix. The objective of backstepping adaptive control is to design a control law such that the state x_1 of system can track the desired value x_1^d .

3 PID control law design

The structure of PID control is showed as following Figure 1. The system is constructed by PID controller and control object. And the object is controller by PID controller which is consisted by proportional item, differential item and integral item.

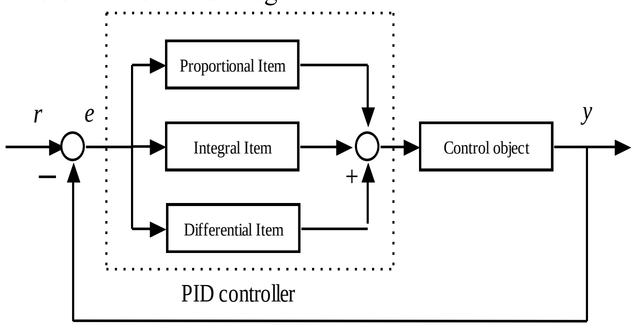

FIGURE 1 Structure of PID control system

PID controller is a kind linear controller, it is composed by the error signal defined by the difference between the desired value x_1^d and output of system x_1 as follows:

$$e(t) = x_1 - x_1^d\,. \tag{2}$$

The PID control law is designed as:

$$u(t) = k_p\left(e(t) + \frac{1}{T_I}\int_0^t e(t)dt + \frac{T_D e(t)}{dt}\right). \tag{3}$$

It can be written as a transfer function as:

$$G(s) = \frac{U(s)}{E(s)} = k_p\left(1 + \frac{1}{T_I s} + T_D s\right), \tag{4}$$

where k_p the coefficient of proportional is item and T_I is the coefficient of integral item and T_D is the coefficient of differential item.

4 Backstepping adaptive control law

Consider the first subsystem:

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2\,. \tag{5}$$

Without loss of generality, two assumptions are proposed as follows:

Assumption 1: a_{12} is not zero, and its sign is known. To make it simple, here we assume $a_{12} > 0$.

Assumption 2: the desired value x_1^d is a constant and its derivative $\dot{x}_1^d = 0$.

Define a new error variable as $e_1 = x_1 - x_1^d$, then

$$\dot{e}_1 = a_{11}x_1 + a_{12}x_2\,. \tag{6}$$

Use backstepping method and design the desired value of x_2 as x_2^d such that:

$$x_2^d = -k_1 e_1 - \hat{k}_2\,, \tag{7}$$

where $\hat{k}_2$ is an adaptive item which is used to approximate information about a_{11} . Substitute it into equation 6, it holds:

$$\dot{e}_1 = a_{11}e_1 + a_{11}x_1^d + a_{12}e_2 + a_{12}x_2^d\,. \tag{8}$$

It can be written as:

$$\dot{e} = a_{11}e_1 + a_{11}x_1^d + a_{12}e_2 + a_{12}(-k_1 e_1 - \hat{k}_2)\,.$$

To make it easy to read, it can be rewritten as :

$$\dot{e} = (a_{11} - a_{12}k_1)e_1 + a_{11}x_1^d - a_{12}\hat{k}_2 + a_{12}e_2\,.$$

Define

$$\bar{k}_1 = a_{11} - a_{12}k_1\,. \tag{9}$$

It is obvious that there exists a big enough k_1 such that $\bar{k}_1 < 0$.

Define

$$\tilde{k}_2 = a_{11}x_1^d - a_{12}\hat{k}_2\,. \tag{10}$$

Then

$$\dot{\tilde{k}}_2 = -a_{12}\dot{\hat{k}}_2\,. \tag{11}$$

Choose a turning law for assumption of unknown parameter as:

$$\dot{\hat{k}}_2 = k_2 e_1 \cdot \quad (12)$$

Then it can be rewritten as:

$$\dot{e}_1 = \bar{k}_1 e_1 + \tilde{k}_2 + a_{12} e_2 \cdot \quad (13)$$

Choose the first Lyapunov function as

$$V_1 = \frac{1}{2a_{12}k_2} \tilde{k}_2^2 . \quad (14)$$

Then

$$\dot{V}_1 = \frac{1}{a_{12}k_2} \tilde{k}_2 \dot{\tilde{k}}_2 . \quad (15)$$

It can be rewritten:

$$\dot{V}_1 = -\frac{1}{a_{12}k_2} \tilde{k}_2 a_{12} \dot{\hat{k}}_2 \quad ,$$

and, then it can be simplified as:

$$\dot{V}_1 = -\frac{1}{a_{12}k_2} \tilde{k}_2 a_{12} k_2 e_1 = -\tilde{k}_2 e_1 .$$

Consider the second subsystem:

$$\dot{e}_2 = a_{21} x_1 + a_{22} x_2 + u - \dot{x}_2^d . \quad (16)$$

Design an adaptive control law:

$$u = -\hat{a}_{21} x_1 - \hat{a}_{22} x_2 + \dot{x}_2^d - k_3 e_2 - k_4 \int e_2 dt . \quad (17)$$

Define

$$\tilde{a}_{21} = a_{21} - \hat{a}_{21} , \quad (18)$$

$$\tilde{a}_{22} = a_{22} - \hat{a}_{22} . \quad (19)$$

Then

$$\dot{\tilde{a}}_{21} = -\dot{\hat{a}}_{21} , \quad (20)$$

$$\dot{\tilde{a}}_{22} = -\dot{\hat{a}}_{22} . \quad (21)$$

Substitute the control law into equation (16), and then:

$$\dot{e}_2 = \tilde{a}_{21} x_1 + \tilde{a}_{22} x_2 - k_3 e_2 - k_4 \int e_2 dt . \quad (22)$$

Design turning law for assumptions of unknown parameters as:

$$\dot{\hat{a}}_{21} = k_5 e_2 x_1 , \quad (23)$$

$$\dot{\hat{a}}_{22} = k_6 e_2 x_2 . \quad (24)$$

Choose the second Lyapunov function as:

$$V_2 = \frac{1}{2k_5} \tilde{a}_{21}^2 + \frac{1}{2k_6} \tilde{a}_{22}^2 . \quad (25)$$

Then

$$\dot{V}_2 = -e_2 x_1 \tilde{a}_{21} - e_2 x_2 \tilde{a}_{22} . \quad (26)$$

Choose the third Lyapunov function as:

$$V_3 = \frac{1}{2} k_4 \left(\int e_2 dt \right)^2 . \quad (27)$$

Then

$$\dot{V}_3 = k_4 e_2 \int e_2 dt. \quad (28)$$

Choose the fourth Lyapunov function as:

$$V_4 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \quad (29)$$

Then

$$\dot{V}_4 = e_1 \dot{e}_1 + e_2 \dot{e}_2 . \quad (30)$$

Choose a big Lyapunov function for the whole system as:

$$V = V_1 + V_2 + V_3 + V_4 . \quad (31)$$

Then

$$\dot{V} = \bar{k}_1 e_1^2 + a_{12} e_2 e_1 - k_3 e_2^2 . \quad (32)$$

The above equation can be reduced with inequality as

$$\dot{V} \le \bar{k}_1 e_1^2 + \frac{1}{2} a_{12} e_1^2 + \frac{1}{2} a_{12} e_2^2 - k_3 e_2^2 \quad . \quad (33)$$

It is obvious that there exists big enough positive k_1 and k_3, such that:

$$\dot{V} \le 0. \quad (34)$$

Then the system is proved to be stable according to Lyapunov stability theory.

5 Numerical simulation

Since the model parameters of second order system are unknown, so the matrix is uncertain. Also different matrix A means a different system . To make it simple and without loss of generality, The desired value is assumed to be 1 and members of matrix A are assumed to be rand numbers of the interval（-10~10）.

5.1 SIMULATION OF PID CONTROL

Parameters of matrix A are chosen as random numbers in the interval（-10~10）as below:

$a_{11} = 8.7; a_{12} = -3.6; a_{21} = -9.3; a_{22} = 7.$

Choose PID control parameters as $k_p = -10, k_i = -5, k_d = -1$, simulation result is shown as figure 2.

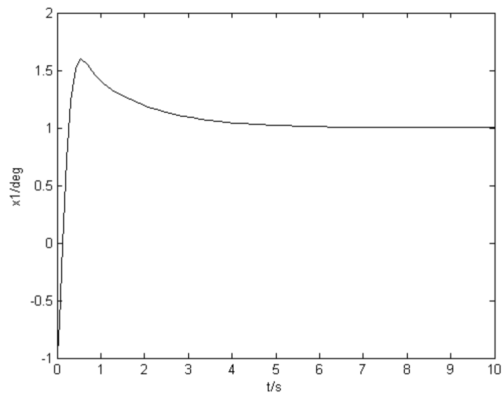

FIGURE 2 The first result of PID control

Continue to use above PID control parameters and use rand function to generate new matrix A six times to do the simulation and results are shown as below Figure 3 to Figure 8.

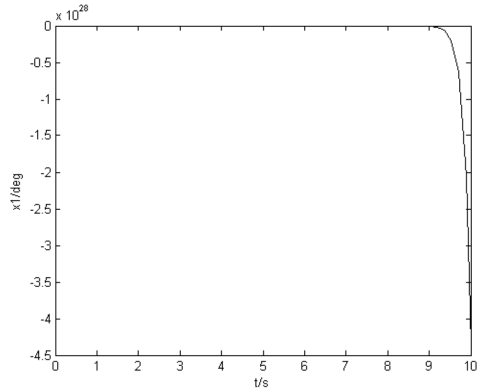

FIGURE 3 The second result of PID control

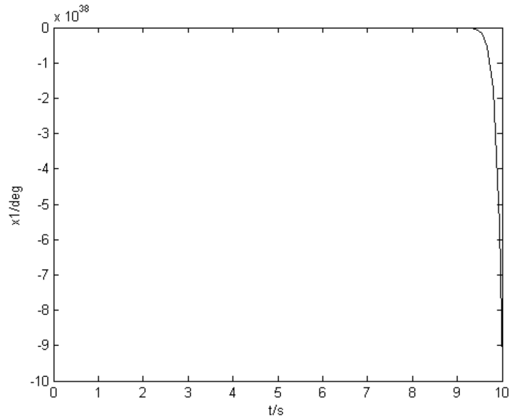

FIGURE 4 The third result of PID control

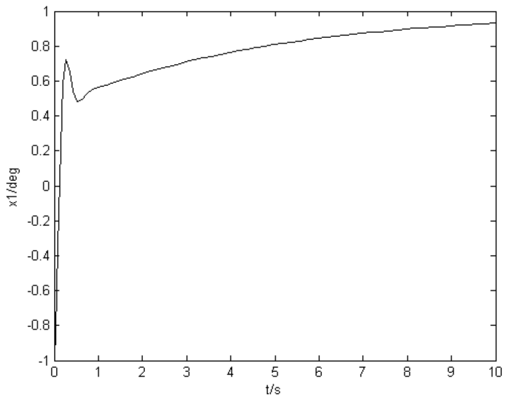

FIGURE 5 The fourth result of PID control

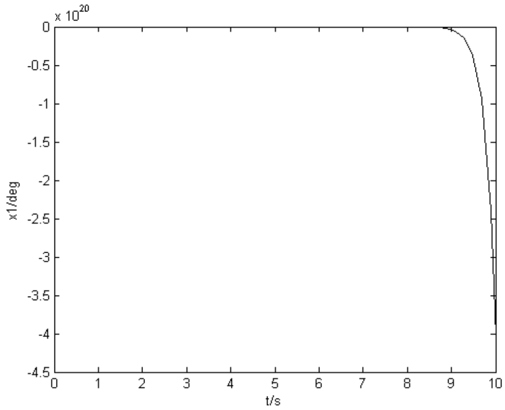

FIGURE 6 The fifth result of PID control

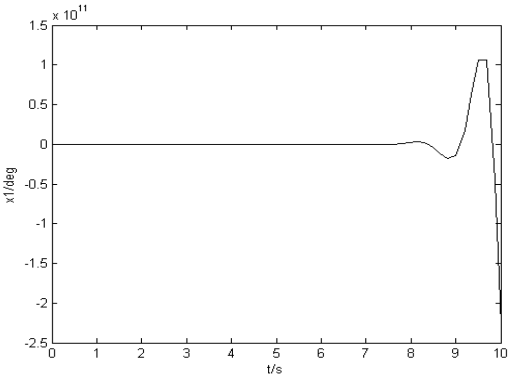

FIGURE 7 The six result of PID control

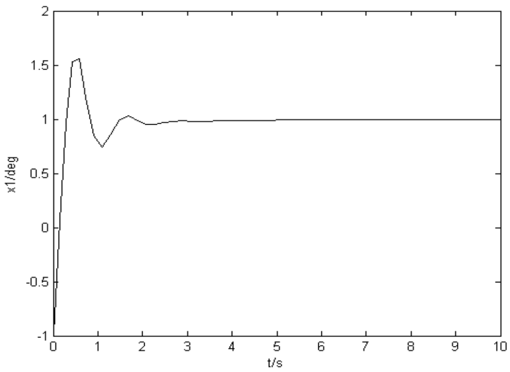

FIGURE 8 The seventh result of PID control

The above simulation shows that the system can not always be stable if the control parameters are random numbers.

5.2 SIMULATION OF BACKSTEPPING CONTROL

Since k_1, k_3 should be chosen big enough to make the system stable, so in the program, parameters are chosen as $k_1 = 30, k_2 = 1, k_3 = 30, k_4 = 1, k_5 = 40, k_6 = 40$. And matrix A is also use the same method as PID to generate. Simulation result are shown as follows:

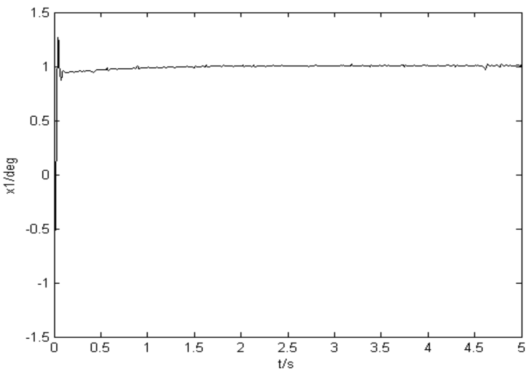

FIGURE 9 The first result of backstepping

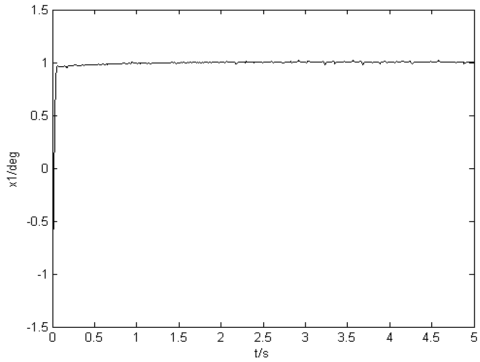

FIGURE 10 The second result of backstepping

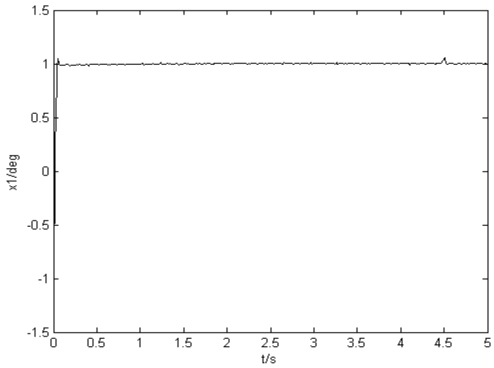

FIGURE 11 The third result of backstepping

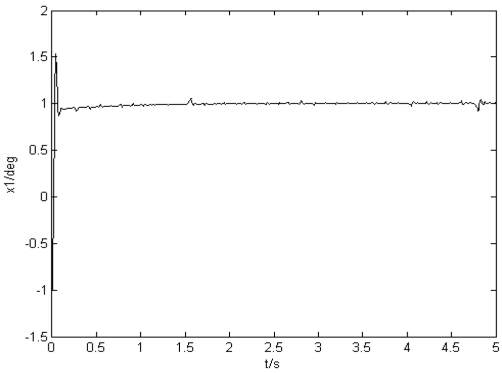

FIGURE 12 The fourth result of backstepping

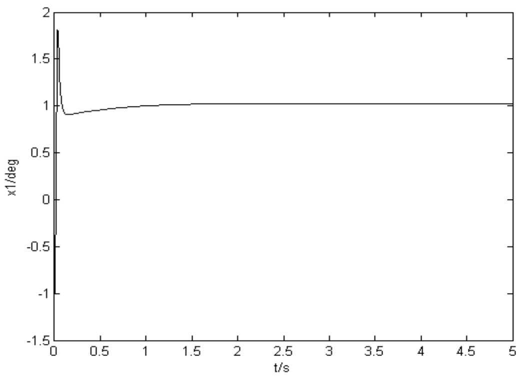

FIGURE 13 The fifth result of backstepping

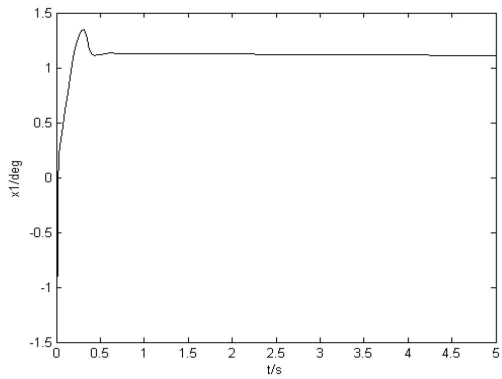

FIGURE 14 The sixth result of backstepping

Simulation result shows that backstepping control is effective for second order system with random uncertainty.

5.3 ANALYSIS OF SIMULATION RESULT

The PID control law is designed based on a certain group of system parameter so it will be unstable if the system parameter changed. Especially, if the system parameter is random, PID control which is mostly designed according to transfer function theory will be not effective enough. And the backstepping method proposed in this paper, which is not depended on the accurate system parameter, is more effective for systems with random parameters.

6 Conclusion

A kind of random second order system with single input coefficient is researched. Both PID and backstepping controller are designed for random system and numerical simulation are done to testify to rightness and effectiveness of the proposed method. Simulation results and comparison of two methods shows that the backstepping methods are more effective than PID method for the situation of second order system with only one input coefficient. Also, the parameters should be chosen from a bound interval randomly. It means that the backstepping method is robust for bounded uncertain parameters.

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Authors

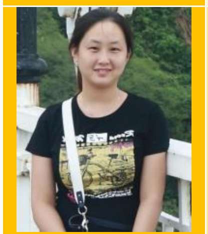
Zhaorui Ma , 14 th February 1978, Huixian City Henan province of China

Current position, grades: a lecturer in the Computer Department of Zhengzhou University of Light Industry (2009).
University studies: Master's degree in computer application technology in 2006 from Huazhong University of Science and Technology, Hubei of China.
Scientific interest: computer application technology and Network control theory
Publications : His typical book named JSP programming technology coursewas published in 2014 in China.
Experience: He graduated from Zhengzhou University of Light Industry , Zhengzhou of China in 2003 and received bachelor's degree with the major of computer science and technology.

Cheng Gan, 11th June 1981,PingXiang JiangXi China

Current position, grades: a lecturer in the Computer Department of Zhengzhou University of Light Industry (2008)
University studies: Master's degree in computer application technology in 2006 from Huazhong University of Science and Technology, Hubei of China.
Scientific interest: the computer application technology and control theory.

Experience: He graduated from Zhengzhou University of Light Industry ,Zhengzhou of China in 2002 and received bachelors degree with the major of computer science and technology.



Na Li , 4 th November 1979, Xinxiang City Henan province of China

Current position, grades: a lecturer in the Computer Department of SIAS International University (2009)
University studies: Master's degree in computer application technology in 2009 from Huazhong University of Science and Technology, Hubei of China.
Scientific interest: the computer application technology and mobile Internet
Publications: Her typical book named JSP programming technology course was published in 2014 in China
Experience: She graduated from Zhengzhou University of Light Industry ,Zhengzhou of China in 2003 and received her bachelor's degree with the major of computer science and technoloy. After that she continued her study in Huazhong University of Science and Technology and received her master's degree in 2009

Xia Li , 25 th August 1962, Jiyuan Henan province of China

Current position, grades: Professor in the Computer Department of Zhengzhou University of Light Industry
University studies: Master's degree in computer application technology in 2002 from Huazhong University of Science and Technology, Hubei of China.
Scientific interest: the computer application technology and mobile Internet
Experience: She graduated from Zhengzhou University,Zhengzhou of China in 1996 and received her bachelor's degree with the major of computer science and technoloy. After that she continued her study in Huazhong University of Science and Technology and received her master's degree in 2002.

Ruiqi Wang, 24th May 1980, Xinxiang City Henan province of China

University studies: received his Doctor degree in Guidance, Navigation and Control in 2011 from Naval Aeronautical and Astronautical University, Yantai of China.
Scientific interest: control theory, missile control and bilateral control
Publications: He has published more than 20 papers, where 7 papers was indexed by Ei.
Experience: He graduated from Naval Aeronautical and Astronautical University, Yantai of China in 2001 and received his bachelor's degree with the major of missile control and test. After that he continued his study in this school and received his master's degree and doctor's degree in 2006 and 2011 respectively.

Junwei Lei , 9th November 1981, Chibi City Hubei province of China

Current position, grades: a lecture of NAAU (2010)
University studies: Doctor degree in Guidance, Navigation and Control in 2010 from Naval Aeronautical and Astronautical University, Yantai of China.
Scientific interest: control theory, chaotic system control, aircraft control and adaptive control
Publications: His typical book named Nussbaum gain control technology of supersonic missiles was published in 2013 in China, more than 60 papers, where 8 papers was indexed by SCI and more than 50 papers was indexed by Ei.
Experience: He graduated from Naval Aeronautical and Astronautical University, Yantai of China in 2003 and received his bachelor's degree with the major of missile control and test. After that he continued his study in this school and received his master's degree and doctor's degree in 2006 and 2010 respectively.